NTPL
ALL INDIA TEST SERIES

CODE - A

TEST ID 001934

JEE (Advanced) - 2019

FULL TEST - 5 (Paper-II)

Time : 3 Hours ***Maximum Marks : 240***

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **60 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question Paper Format :

9. The question paper consists of **3 parts (Part I: Physics, Part II: Chemistry & Part III: Maths)**. Each part has **3 sections**.
10. **Section I** contains **8 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).
11. **Section II** contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which ***ONE OR MORE*** is/are correct.
12. **Section III** contains **2 "paragraph" type questions**. Each paragraph describes an experiment, a situation or a problem. Two multiple choice questions will be asked based on this paragraph. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which ***ONE OR MORE*** is/are correct.

C. Marking Scheme :

13. For each question in **Section I**, you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.
14. For each question in **Section II**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **minus one (-2) marks** will be awarded.
15. For each question in **Section III**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **minus one (-2) marks** will be awarded.

Name of the Candidate (in Capitals) ______________________________

Test Centre ____________________ Centre Code ____________________

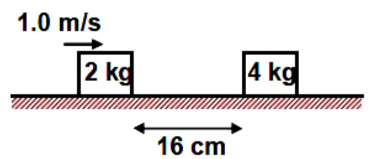
Candidate's Signature ____________________ Invigilator's Signature ____________________

PART I : PHYSICS

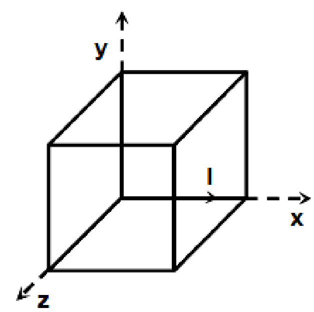
SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened.
  0 In all other cases.

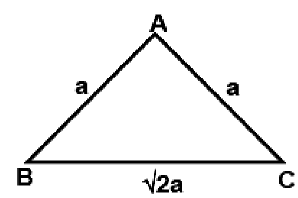
1. The friction coefficient between the horizontal surface and each of the block shown in the figure is 0.2. The collision between the blocks is perfectly elastic. Find the separation (in cm) between them when they come to rest. Take $g = 10\text{m/s}^2$.



2. A cubical frame having edge length a, each having a resistance R is placed in a gravity free space such that axes are along sides of cube. Now a cylindrical region of time varying magnetic field is created around the cube having axis in direction of vector $\left(\hat{i}+\hat{j}+\hat{k}\right)$. It was found that current through edge along x axis is $I = \left|\eta \frac{a^2}{R}\frac{dB}{dt}\right|$, where B is the instantaneous magnetic field. What should be the value of η?

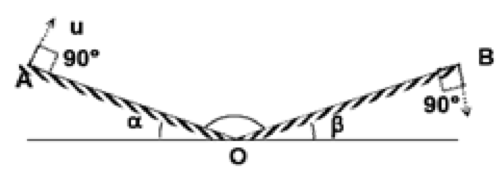


3. ABC is an isosceles triangular sheet of negligible thickness, made of isotropic material. Mass of the sheet is m. Its moment of inertia about axes perpendicular to its plane and passing through the points A and B are I_1 and I_2 respectively. [$AB = AC = a$ and $BC = \sqrt{2}a$]. Find I_2/I_1.



4. Two inclined planes OA and OB of inclinations to the horizontal are α and β, each equal to 30° as shown in the figure. A particle is projected at an angle of 90° with plane OA from point A and its strikes the plane OB at point B normally. Then find the speed of projection in m/s.

(given that $OA = OB = 20$ cm and $g = 10\text{ m/s}^2$)



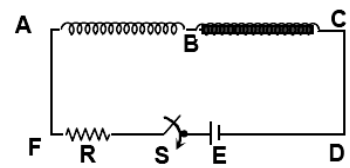
5. If the sound heard by observer, whose equation is given as $y = 8\sin 10\pi t \cos 200\pi t$ at $x = 0$

The number of beat frequency heard by observer is $2k$, then find the value of k.

6. A certain amount of a mono-atomic ideal gas undergoes a process $\rho u^{\eta} = C$, where ρ is the density of the gas and u is the internal energy of the gas. It was found that the ratio $r = \dfrac{\Delta W}{\Delta Q}$ for the process was $r = 2/3$. What is the value of η?

7. Current in a X-ray tube operating at 40 kV is 10 mA. 1% of the total kinetic energy of electrons hitting the target is converted into X-rays. Then the heat produced in the target per second is 99n Joule

8. Two ideal solenoids of same dimensions. One is air cored with 600 turns while other is Aluminium cored with 200 turns (relative permeability of Aluminium is 3), are connected in a circuit as shown in the figure. The switch S is closed at t = 0. Find the ratio of potential difference across air-cored solenoid to that of Aluminium cored solenoid at any time t.



SECTION 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are)correct.
- For each question, darken the bubble(s) corresponding to all the correct options(s) in the ORS.
- Marking scheme:

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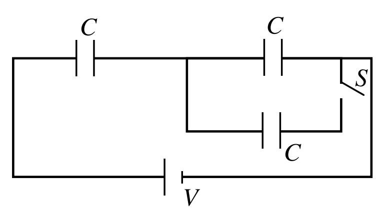
0 If none of the bubbles is darkened

–2 In all other cases

9. In the given circuit diagram on closing the switch S



(a) work done by battery is $\frac{1}{2}CV^2$

(b) Heat generated in the circuit is $\frac{1}{4}CV^2$

(c) work done by batter is $\frac{1}{6}CV^2$

(d) heat generated in the circuit is $\frac{1}{12}CV^2$

10. A particle of mass m and charge q is projected in a region where an electric field is existing and given by $\vec{E} = E_0\hat{i}$, with a velocity $v_0\hat{j}$ from the origin at time t = 0, then choose the correct statements (assuming $m^2v_0^2 = 2qE_0mx_0$).

(a) radius of curvature of the particle when its x-coordiante becomes x_0 is $2x_0$.

(b) radius of curvature of the particle when its x-coordiante becomes x_0 is $4\sqrt{2}\,x_0$.

(c) speed of the particle when its x-coordinate becomes x_0 is $\sqrt{2}v_0$.

(d) speed of the particle when its x-coordinate becomes x_0 is $2v_0$.

11. Two particles, P of mass $2m$ and Q of mass m, are subjected to mutual force of attraction and no other force acts on them. At $t = 0$, P is at rest at point O and Q is moving away from O with a speed $5u$. At a later instant $t = T$ (before any collision has taken place), Q is moving towards O with speed u. Then

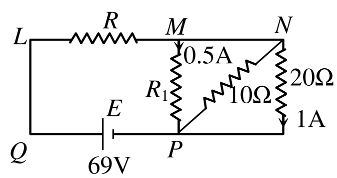
(a) momentum of particle P at $t = T$ is zero.

(b) momentum of particle P at $t = T$ is 6 mu.

(c) work done by the force of attraction during $0 \le t \le T$ is 12 mu^2.

(d) work done by the force of attraction during $0 \le t \le T$ is –3 mu^2.

12. In the circuit shown in the figure.



(a) the current through NP is 0.5 A

(b) the value of R_1 = 40 Ω

(c) the value of R = 14 Ω

(d) the potential difference across R = 49 V

13. A charged particle moves in a gravity-free space without change in velocity. Which of the following is/are possible?

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$

14. A positive charge is passing through an electromagnetic field in which $\vec{E}$ & $\vec{B}$ are directed towards y-axis & z-axis respectively. If a charge particle passes through the region undeviated, then its velocity is/are represented by (here a, b & c are constant)

(a) $\vec{v} = \frac{E}{B}\hat{i} + a\hat{j}$ (b) $\vec{v} = \frac{E}{B}\hat{i} + b\hat{k}$ (c) $\vec{v} = \frac{E}{B}\hat{i} + c\hat{i}$ (d) $\vec{v} = \frac{E}{B}\hat{i}$

15. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to the left with a speed v. The speed of sound in the medium is c. Then

(a) the number of waves striking the surface per second is $\frac{f(c+v)}{c}$

(b) the wavelength of the reflected wave is $\frac{c(c-v)}{f(c+v)}$

(c) the wavelength of the reflected wave is $\frac{c(c+v)}{f(c-v)}$

(d) the number of beats heard by the stationary observer to the left of the reflecting surface is $\frac{2vf}{c-v}$.

16. If the wavelength of light in an experiment on photo electric effect is doubled

(a) the photoelectric emission will not take place.

(b) the photoemission may or may not take place

(c) the stopping potential will increase

(d) the stopping potential will decrease under the condition that energy of photon of doubled wavelength is more than work function of metal

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:

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  0 If none of the bubbles is darkened

  –2 In all other cases

PARAGRAPH 1

Net force on a current carrying loop kept in uniform magnetic field is zero and the torque on the loop $\vec{\tau} = \vec{M} \times \vec{B}$ where M and B are magnetic dipole moment and magnetic field intensity respectively. If it is free to rotate then it will rotates about an axis passing through its centre of mass and parallel to $\vec{\tau}$. Potential energy of the loop is given by $U = -\vec{M}.\vec{B}$. Assume a current carrying ring with its centre at the origin and having moment of inertia 2×10^{-2} kg– m^2 about an axis passing through one of its diameter and magnetic moment $\vec{M} = (3\hat{i} - 4\hat{j})$ Am^2. At time $t = 0$ a magnetic field $\vec{B} = (4\hat{i} + 3\hat{j})T$ is switched on. Then

17. Torque acting on the loop is

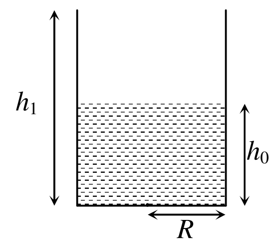
(a) zero (b) $25\hat{k}$ Nm (c) $16\hat{k}$ Nm (d) $10\hat{k}$ Nm

18. Angular acceleration of the ring at time $t = 0$ (in rad/s^2) is

(a) 5000 (b) 1250 (c) 2500 (d) zero

PARAGRAPH 2

A cylindrical vessel of radius 1m and height 3m is filled with an ideal liquid upto a height of 2m as shown in figure. The cylinder is rotated about its axis with angular velocity ω.



19. The maximum value of angular velocity of the cylindrical vessel so that the liquid will not start spilling over the brim is

(a) $2\sqrt{5}$ rad/s (b) $2\sqrt{10}$ rad/s (c) $2\sqrt{15}$ rad/s (d) $3\sqrt{10}$ rad/s

20. If liquid is not spilling over the brim, the bottom most point of the liquid meniscus will be at a height of h from the base of the cylindrical vessel. The value of h is

(a) 2 m (b) 1 m (c) 1.5 m (d) 1.2 m

PART II : CHEMISTRY

SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:
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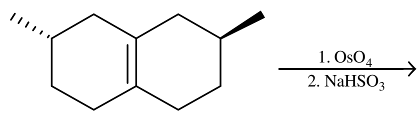
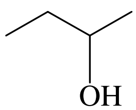
21. How many products will be formed in this reaction?



22. 

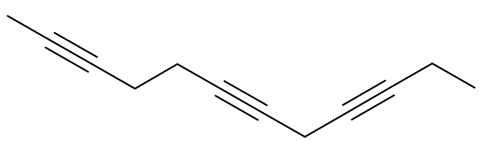
$\xrightarrow[\Delta]{H^{\oplus}}$ Possible products (A) $\xrightarrow{Br_2/CCl_4}$ (B) products. The (A+B) is

23. (a) $\underset{(all\ isomers)}{C_2FClBrI} \xrightarrow[Ni]{H_2} (A)$ (exclude stereoisomer)

(b) $\underset{(all\ isomers)}{C_4H_8} \xrightarrow[H_2/Ni]{} (B)$ (exclude stereoisomers)

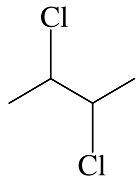
The $(A+B)$ is

24. How many compounds are formed upon the ozonolysis of the following triyne?



25. How many isomers of C_8H_{10} when reacts with hot alkaline $KMnO_4$ give di-carboxylic acid as a product?

26. Reaction – 1: $Ph-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow{PCl_5} \xrightarrow{\boxed{a}NaNH_2} \xrightarrow{CH_3I} Ph-C \equiv C-CH_3$

Reaction – 2:  $\xrightarrow{\boxed{b}NaNH_2} CH_3-C \equiv C-CH_3$

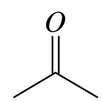
Reaction – 3: $\underset{Cl}{Ph-\underset{|}{CH}}-\underset{Cl}{\underset{|}{CH_2}} \xrightarrow{\boxed{c}NaNH_2} \xrightarrow{PhCH_2I}$

$Ph-C \equiv C-CH_2-Ph$

a, b, c are moles used. Sum of $[a+b+c]=$

27. Amongst the following, the total number of compounds which will undergo, addision reaction will be:

(a) $CH_3-CH=CH_2$

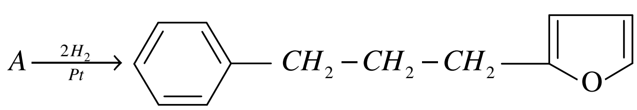
(b) 

(c) $CH_3-C \equiv N$

(d) $CH_3-\overset{O}{\overset{\|}{C}}-H$

(e) CH_4

(f) $CH_3C \equiv CCH_3$

28. 

Unit of unsaturation incompound (A)?

SECTION 2 (Maximum Marks : 32)

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  –2 In all other cases

29. The function of enzymes in the living system is to

(a) Transport oxygen
(b) Provide immunity
(c) Catalyse biochemical reaction
(d) Provide energy

30. The deficiency of vitamin K causes

(a) Haemorrhage
(b) Lenghening time of blood clotting
(c) Inflammation of tung
(d) Both (1) and (2)

31. Which of the following show amphoteric behaviour:

(a) $Zn(OH)_2$
(b) BeO
(c) Al_2O_3
(d) $Pb(OH)_2$

32. Which of the following statements are correct:

(a) F is the most electronegative and Cs is the most electropositive element.

(b) The electronegativity of halogens decreases from F to I

(c) The electron affinity of Cl is higher than that of F though their electronegativities are in the reverse order

(d) The electron affinity of noble gases is almost zero.

33. Which are correct match

(a) O > F > N > C — IInd I.P.

(b) $S^{-2} > Cl^{-} > K^{+} > Ca^{+2}$ – Ionic radius

(c) N > C > P > Si – E. N.

(d) F > Na > Ne — I^{st} I.P.

34. Consider the following ionization steps:

$$M(g) \longrightarrow M^{+}(g) + e^{-}; \Delta H = 100\, ev$$

$$M(g) \longrightarrow M^{2+}(g) + 2e^{-}; \Delta H = 250\, eV$$

select correct statement(s) :

(a) $I.E._1$ of M (g) is 100 eV
(b) $I.E._1$ of M^{+} (g) is 150 eV
(c) $I.E._2$ of M(g) is 250 eV
(d) $I.E._2$ of M (g) is 150 eV

35. Isopropylbenzene can be prepared by:

(a) Benzene $+CH_3CH=CH_2 \xrightarrow{H_2SO_4}$

(b) Benzene $+CH_3-\underset{\substack{|\\Cl}}{CH}-CH_3 \xrightarrow{H_2SO_4}$

(c) Benzene $+CH_3\underset{\substack{|\\Cl}}{CH}-CH_3 \xrightarrow{AlCl_3}$

(d) Benzene $+CH_3CH_2CH_2Cl \xrightarrow[\Delta]{AlCl_3}$

36. Which of the following characteristic does an aromatic compound exhibit?

(a) It should have (4n +2) π-electrons in the ring

(b) It should be planar and conjugated

(c) It should have 4n $\pi-$electrons in the ring

(d) It should possess high resonance energy

SECTION 3 (Maximum Marks : 16)

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  –2 In all other cases

PARAGRAPH 1

342 g of 20% by mass of $Ba(OH)_2$ solution (sp. gr. 0.57) is reacted with 200 mL of 2 M HNO_3 according to given balanced reaction :

$$Ba(OH)_2 + 2HNO_3 \rightarrow Ba(NO_3)_2 + 2H_2O$$

37. The nature of the final solution is:

(a) acidic (b) neutral (c) basic (d) can't say

38. Find the molarity of the ion in resulting solution by which nature of the above solution is identified, is

(a) 0.5 M (b) 0.8 M (c) 0.4 M (d) 1 M

PARAGRAPH 2

An ideal gas, having ratio of specific heat γ undergoes a process in which its internal energy relates to the volume as $U = \alpha\sqrt{V}$, where α is a constant. If the gas is expanded from volume V_1 to V_2.

39. The work per formed by gas is:

(a) $2\alpha(\gamma-1)\left[\sqrt{V_2}-\sqrt{V_1}\right]$ (b) $\alpha(\gamma-1)\left[\sqrt{V_2}-\sqrt{V_1}\right]$

(c) $2\alpha(\gamma-1)[V_2-V_1]$ (d) $\alpha(\gamma-1)[V_2-V_1]$

40. If the ideal gas is diatomic and its increase in internal energy is 100 J then the work performed by gas is : (Ignore vibrational degree of freedom)

(a) 80 J (b) 180 J (c) 100 J (d) 20 J

PART III : MATHS

SECTION 1 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened.
  0 In all other cases.

41. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the greatest positive integer in the range of $\frac{3\left|\vec{a}+\vec{b}\right|}{2}+2\left|\vec{a}-\vec{b}\right|$.

42. The value of $2^{100}\sin\left(\frac{\pi}{100}\right)\sin\left(\frac{2\pi}{100}\right)...\sin\left(\frac{99}{100}\pi\right)$ is k. Then value of $\frac{k}{50}$ is …….. .

43. $\sum_{k=1}^{360}\left\{1/\left[k\sqrt{k+1}+(k+1)\sqrt{k}\right]\right\}$ is the ratio of two relative prime positive integers m and n. The value of $|m-n|$ is …….. .

44. The coefficient of x^{50} in the polynomials after parenthesis have been removed and like terms have been collected in the expansion $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+...+x^{1000}$ is $\frac{\lambda!}{\mu!\nu!}$, then the value of $\left[\frac{\lambda+2\mu+3\nu}{400}\right]$ must be $(\nu>\mu)$ …….. .

45. A ray emanating from the point (–3, 0) is incident on the ellipse $16x^2+25y^2=400$ at the point P with ordinate 4. If the equation of the reflected ray after first reflection is $4x+3y=\lambda$, then the value of $2^{\lambda-10}$ must be …….. .

46. The number of points of discontinuity of $f(x)=\lim_{n\to\infty}\left[\frac{(x^{2n}-1)}{(x^{2n}+1)}\right]$ is …….. .

47. If $f\left[\frac{(x+y)}{3}\right]=\frac{[2+f(x)+f(y)]}{3}$ for all real x and y and $f'(2)=2,$ then $f(3)$ is equal to …….. .

48. If $\int \frac{\left(x-x^3\right)^{1/3}}{x^4}dx = a\left(\frac{1}{x^2}-1\right)^b + C,$ then the value of $\frac{1}{|ab|}$ is …….. .

SECTION 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions.
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- For each question, darken the bubble(s) corresponding to all the correct options(s) in the ORS.
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  0 If none of the bubbles is darkened

  –2 In all other cases

49. Let $f(x)=x^2+xg'(1)+g''(2)$ and $g(x)=x^2+x\,f'(2)+f''(3),$ then :

(a) $f'(1)=4+f'(2)$ (b) $g'(2)=8+g'(1)$ (c) $g''(2)+f''(3)=4$ (d) none of these

50. If α and β values of x which satisfy the equation $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x,$ then :

(a) $\alpha+\beta=\frac{1}{2}$ (b) $\alpha\beta=0$ (c) $\alpha+\beta=1$ (d) $\alpha\beta=1/2$

51. If $0<a<b<c$ and the roots $\alpha,\ \beta$ of the equation $ax^2+bx+c=0$ are non-real complex roots, then :

(a) $|\alpha|=|\beta|$ (b) $|\alpha|>1$ (c) $|\beta|<1$ (d) none of these

52. If $(a\cos\theta_1,\ a\sin\theta_1),\ (a\cos\theta_2,\ a\sin\theta_2)$ and $(a\cos\theta_3,\ a\sin\theta_3)$ represents the vertices of an equilateral triangle inscribed in a circle, then :

(a) $\cos\theta_1+\cos\theta_2+\cos\theta_3=0$ (b) $\sin\theta_1+\sin\theta_2+\sin\theta_3=0$

(c) $\tan\theta_1+\tan\theta_2+\tan\theta_3=0$ (d) $\cot\theta_1+\cot\theta_2+\cot\theta_3=0$

53. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola, then a point of intersection of the circle and the parabola is :

(a) $\left(\frac{p}{2}, p\right)$ (b) $\left(\frac{p}{2}, -p\right)$ (c) $\left(-\frac{p}{2}, p\right)$ (d) $\left(-\frac{p}{2}, -p\right)$

54. A curve $g(x) = \int x^{27}\left(1+x+x^2\right)^6\left(6x^2+5x+4\right)dx$ is passing through origin, then :

(a) $g(1) = \frac{3^7}{7}$ (b) $g(1) = \frac{2^7}{7}$ (c) $g(-1) = \frac{1}{7}$ (d) $g(-1) = \frac{3^7}{14}$

55. The value(s) of 'a' for which the area of the triangle included between the axes and any tangent to the curve $x^a y = \lambda^a$ is constant is/are :

(a) $-\frac{1}{2}$ (b) –1 (c) $\frac{1}{2}$ (d) 1

56. Equation of the curve in which the subnormal is twice the square of the ordinate is given by

(a) $\ln y = 2x + \ln c$ (b) $y = ce^{2x}$ (c) $\ln y = 2x - \ln c$ (d) none of these

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:

  +4 If the bubble(s) corresponding to all the correct options(s) is(are) darkened.

  0 If none of the bubbles is darkened

  –2 In all other cases

PARAGRAPH 1

If $D^* f(x) = \lim_{h\to 0} \frac{f^2(x+h) - f^2(x)}{h}$ where $f^2(x) = \{f(x)\}^2$

57. If $u = f(x),\ v = g(x),$ then the value of $D^*(u \cdot v)$ is :

(a) $(D^*u)v + (D^*v)u$ (b) $u^2 D^* v + v^2 D^* u$ (c) $D^* u + D^* v$ (d) $uvD^*(u+v)$

58. The value of $D^* f(x)$ at the point on the curve $y = f(x)$ such that tangent to it are parallel to x-axis, then :

(a) $f(x)$ (b) zero (c) $2f(x)$ (d) $xf(x)$

PARAGRAPH 2

The probability that a family has exactly n children is αp^n, $n \geq 1$. All sex distributions of n children in a family have the same probability.

59. The probability that a family contains exactly k boys is (where $k \geq 1$) :

(a) $\alpha p^k (1-p)^{-k-1}$ (b) $2\alpha p^k (2-p)^{-k-1}$ (c) $2\alpha p^k (2-p)^{-k}$ (d) $2\alpha p^{k-1} (2-p)^{-k-1}$

60. The probability that a family includes at least one boy is :

(a) $\dfrac{\alpha^2 p}{(2-p)(1-p)}$ (b) $\dfrac{\alpha p^2}{(2-p)(1-p)}$ (c) $\dfrac{\alpha p}{(2-p)(1-p)}$ (d) $\dfrac{2\alpha p}{(2-p)(1-p)}$

SOLUTION OF AITS JEE (ADVANCED) FULL TEST – 5

PHYSICS

1. **(5)**

Velocity of first block before collision, $v_1^2 = 1^2 - 2(2)\times 0.16 = 1 - 0.64$

$v_1 = 0.6$ m/s

By conservation of momentum, $2 \times 0.6 = 2v_1' + 4v_2'$

also $v_2' - v_1' = v_1$ for elastic collision

It gives $v_2' = 0.4$ m/s

$v_1' = -0.2$ m/s

Now distance moved after collision

$$s_1 = \frac{(0.4)^2}{2\times 2} \ \& \ s_2 = \frac{(0.2)^2}{2\times 2}$$

$\therefore\ s = s_1 + s_2 = 0.05$ m = 5 cm.

2. **(0)**

By symmetry, no current will flow in arm along x-axis.

3. **(2)**

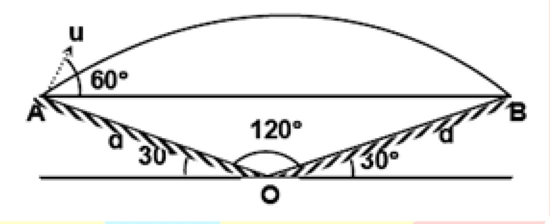
4. **(2)**

Since $\alpha = \beta$

$AO = BO = 20$ cm

$AB = \sqrt{d^2 + d^2 - 2d\cdot d\cos 120^\circ}$

$AB = \sqrt{3}d$



Now AB is the range of the projectile from A to B.

$$\sqrt{3}d = \frac{u^2\sin 2\theta}{g} \Rightarrow \sqrt{3}d = \frac{u^2\sqrt{3}}{2g}$$

Putting $d = 0.2$ m

$u = 2$ m/s

5. **(5)**

6. **(3)**

$\rho u^\eta = C$

$\Rightarrow V^{-1}T^\eta = C \ \Rightarrow TV^{-1/\eta} = C \ \Rightarrow PV^{1-(1/\eta)} = C$

Compare with $PV^x = C \Rightarrow x = 1 - \frac{1}{\eta}$

Now, $\Delta Q \propto \left(\frac{R}{r-1}+\frac{R}{1-x}\right)$ and $\Delta W \propto \left(\frac{R}{1-x}\right)$

$$\Rightarrow \frac{\Delta W}{\Delta Q}=\frac{\left(\frac{R}{1-x}\right)}{\frac{R}{r-1}+\frac{R}{1-x}}=\left(\frac{r-1}{r-x}\right)=\frac{2}{3}=\frac{\frac{5}{3}-1}{\frac{5}{3}-x}=\frac{2}{3}\Rightarrow x=2/3=1-\frac{1}{\eta}\Rightarrow \eta=3$$

7. **(4)**

Power = 40 kV 10 mA = 400 W.

99 % of it is produced as heating power

Heating power = 396 W = 396 Joules per second.

8. **(3)**

$$L_1=\frac{\mu_0 N_1^2 A}{\ell},\ L_2=\frac{\mu_0 N_2^2 A\mu_r}{\ell}$$

Since $V=-L\frac{dl}{dt}$, So

$$\frac{V_1}{V_2}=\frac{L_1}{L_2}=\left(\frac{N_1}{N_2}\right)^2\frac{1}{\mu_r}=\left(\frac{600}{200}\right)^2\times\frac{1}{3}=3$$

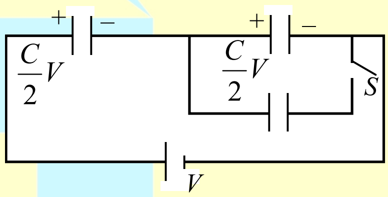
9. **(c), (d)**

On closing the switch work done battery is

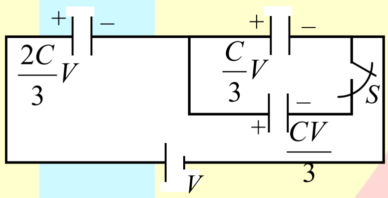
$$=V\left(\frac{2CV}{3}-\frac{CV}{2}\right)=\frac{1}{6}CV^2$$

Heat generated $=\frac{1}{6}CV^2-\left[\left\{\frac{1}{2}\frac{2C}{3}V^2-\frac{1}{2}\frac{C}{2}V^2\right\}\right]$

$$=\frac{1}{12}CV^2$$



Before closing the switch



After closing switch

10. **(b), (c)**

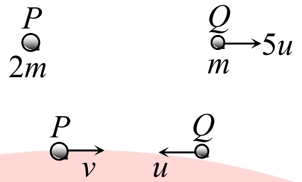
$$v^2=v_0{}^2+2\left(\frac{qB_0}{m}\right)x_0$$

$$v=\sqrt{2}v_0$$

$$a_n=\frac{qE_0}{m}\frac{v_0}{\sqrt{v_x^2+v_0^2}}$$

$$R=\frac{v^2}{a_n}=\frac{[m^2v_0^2+2qE_0mx_0]^{\frac{3}{2}}}{qE_0v_0m^2}=4\sqrt{2}\,x_0$$

11. **(b) and (d)**



$$5mu = 2mv - mu$$

$$v = 3u$$

$$\frac{1}{2}m(5u)^2 + W = \frac{1}{2}mu^2 + \frac{1}{2}\times 2mv^2$$

$$W = -3mu^2$$

12. **(b), (c), (d)**

Current across NP, $I_{NP}\times 10 = 20\times 1$ or $I_{NP} = 2A$

Across MP, $0.5R_1 = 20$ or $R_1 = 40\Omega$

Total current $= 2 + 0.5 + 1.0 = 3.5\Omega$

$3.5 = \dfrac{69}{R + 40/4}$ yields $R = 4\Omega$

13. **(a), (b), (d)**

14. **(b), (d)**

In both case (b) and (d), $\vec{F}_{net} = 0$ so it passes the region undeviated.

15. **(a), (b), (d)**

Number of waves striking the surface per unit time is given by

$$n = \frac{\text{distance travelled per unit time}}{\text{wavelength}} = \frac{c+v}{\lambda} = f\left(1 + \frac{v}{c}\right)$$

The wavelength of the reflected wave is given by

$$\lambda' = \frac{c}{f'} = \frac{c}{f}\left(\frac{c-v}{c+v}\right)$$

Beat frequency $= \dfrac{f\left(1+\dfrac{v}{c}\right)}{1-\dfrac{v}{c}} - f = \dfrac{2fv}{c-v}$

16. **(b), (d)**

Photo-electric emission is possible if $\lambda < \lambda_0$ where λ_0 is threshold wavelength

$$\frac{hc}{\lambda} = e|V_s| + \phi$$

As λ increases, V_s decreases.

17. **(b)**

$$\vec{\tau} = \vec{M}\times\vec{B} = \left(3\hat{i} - 4\hat{j}\right)\times\left(4\hat{i} + 3\hat{j}\right) = 25\hat{k}\ \text{N}-\text{m}$$

18. **(b)**

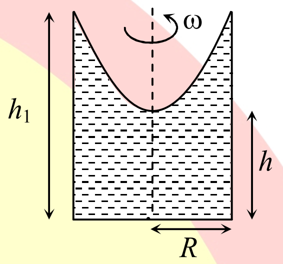
$$\tau = I\alpha \quad \Rightarrow \alpha = \left(\frac{\tau}{I}\right) = \frac{25}{0.02} = 1250\,\text{rad/s}^2$$

19. **(b)**

The shape of the meniscus of the liquid surface will be a parabola. Considering vertex of parabola as origin, the shape of meniscus will be given by the equation

$$y = \frac{\omega^2 x^2}{2g}$$

Since, the liquid is not spilling, the total volume remains conserved.



$$\pi R^2 h_0 = \pi R^2 h + \frac{1}{2}\pi R^2 (h_1 - h) \qquad \ldots\text{(i)}$$

and $$h_1 - h = \frac{\omega^2 R^2}{2g} \qquad \ldots\text{(ii)}$$

From (i) and (ii),

$$\omega = \sqrt{40} = 2\sqrt{10}\ \text{rad/s}$$

20. **(b)**

$$\frac{h_1 + h}{2} = h_0 \Rightarrow h = 1\text{m}$$

CHEMISTRY

21. **(2)**	22. **(8)**	23. **(5)**	24. **(4)**
25. **(3)**	26. **(8)**	27. **(5)**	28. **(9)**
29. **(c)**	30. **(d)**	31. **(a), (b), (c), (d)**	32. **(a), (b), (c), (d)**
33. **(a), (b), (c)**	34. **(a), (b), (d)**	35. **(a), (c), (d)**	36. **(a), (b), (d**
37. **(c)**	38. **(a)**	39. **(a)**	40. **(a)**

MATHS

41. **(5)**

Let angle between $\vec{a}$ and $\vec{b}$ be θ

We have $|\vec{a}| = |\vec{b}| = 1$

Now $|\vec{a} + \vec{b}| = 2\cos\theta/2$ and $|\vec{a} - \vec{b}| = 2\sin\theta/2$

Consider $F(\theta) = \frac{3}{2}\left(2\cos\theta/2\right) + 2\left(2\sin\theta/2\right) = 3\cos\theta/2 + 4\sin\theta/2$

$\therefore\ F(\theta) = 3\cos\theta/2 + 4\sin\theta/2,\ \theta \in [0, \pi]$

greatest +ve integer = 5

42. **(2)**

100^{th} roots of unit are $\alpha_0, \alpha_1, ...\alpha_{99}$

$$\alpha_j = \cos\left(\frac{2\pi j}{100}\right) + i\sin\left(\frac{2\pi j}{100}\right)$$

Thus putting $x = 1$

$$(1-\alpha_1)(1-\alpha_2)..........(1-\alpha_{99}) = 100$$

$$\Rightarrow (1-\bar{\alpha}_1)\,(1-\bar{\alpha}_2)...........(1-\bar{\alpha}_{99}) = 100$$

$$\therefore\ (1-\alpha_1)(1-\bar{\alpha}_1)(1-\alpha_2)(1-\bar{\alpha}_2)......(1-\alpha_{99})\quad(1-\bar{\alpha}_{99}) = 10^4$$

$$(1-\alpha_k)(1-\bar{\alpha}_k) = 2\left(1-\cos\frac{2k\pi}{100}\right) = 2^2\sin^2\frac{k\pi}{100}$$

$$2^{198}\sin^2 \pi/100 ....\ \sin^2\left(\frac{99\pi}{100}\right) = 10^4$$

$$2^{99}\sin\left(\frac{\pi}{100}\right).................\sin\left(\frac{99\pi}{100}\right) = 100$$

$k = 100 \qquad\qquad k/50 = 2$

43. **(1)**

$$T_k = \frac{1}{\sqrt{k}\sqrt{k+1}\left[\sqrt{k}+\sqrt{k+1}\right]} = \frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k}\sqrt{k+1}} = \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}$$

$$S = \sum_{k=1}^{360}\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}} \quad = 1 - \frac{1}{\sqrt{361}} = 1 - \frac{1}{19} = \frac{18}{19}$$

44. **(9)**

$$(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + ...x^{1000}$$

$$= \frac{(1+x)^{1000}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]}{\left[1-\left(\frac{x}{1+x}\right)\right]} = \frac{(1+x)^{1000} - \frac{x^{1001}}{1+x}}{\frac{\cancel{x}+1-\cancel{x}}{1+x}} = (1+x)^{1001} - x^{1001}$$

Hence the coefficient of $x^{50} = {}^{1001}C_{50} = \dfrac{1001!}{50!\,951!}$

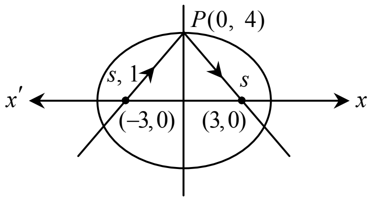
45. **(4)**

For point P, y co-ordinate = 4

$\because$ Given ellipse is $16x^2 + 25y^2 = 400 \Rightarrow 16x^2 + 25(4)^2 = 400$

$P(0, 4)$

$e^2 = 1 - \frac{16}{25} = \frac{9}{25}$ foci $(\pm ae, 0)$ i.e. $(\pm 3, 0)$



Equation of reflected ray (i.e. P.S.) is

$\frac{x}{3} + \frac{y}{4} = 1$

$4x + 3y = 12$

$\lambda = 12 \quad 2^{\lambda - 10} = 2^{12-10} = 4$

46. **(2)**

$f(x) = \lim_{n \to \infty} \frac{(x^2)^n - 1}{(x^2)^n + 1} = \lim_{n \to \infty} \frac{1 - \frac{1}{(x^2)^n}}{1 + \frac{1}{(x^2)^n}}$

$= -1, \ 0 \le x^2 < 1$

$0, \ x^2 = 1$

$1, \ x^2 > 1$

$= 1, \ x^2 < -1$

$0, \ x = -1$

$-1, \ -1 < x < 1$

$0, \ x = 1$

$1, \ x > 1$

$f(x)$ is discontinuous at $x = \pm 1$

47. **(8)**

$f\left(\frac{x+y}{3}\right) = \frac{2 + f(x) + f(y)}{3}$

Replacing x by $3x$ and y by O, we get

$f(0) = 2$

$f(x) = \frac{2 + f(3x) + f(0)}{3}$

$f(3x) - 3f(x) + 2 = -f(0)$

When $x = 0$ and $y = 0$ in eq (1) we get

$f(0) = 2$

Now $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f\left[\frac{(3x+3h)}{3}\right]-f(x)}{h}$

$$=\lim_{h\to 0}\frac{\frac{2+f(3x)+f(3h)-f(x)}{3}}{h}=\lim_{h\to 0}\frac{f(3x)-3f(x)+f(3h)+2}{3h}$$

$$=\lim_{h\to 0}\frac{f(3h)-f(0)}{3h}\Rightarrow f'(0)=c \text{ there } f'(x)=c$$

At $x=2,\ f'(2)=c=2$

$f'(x)=2$

Integrating both sides we get

$f(x)=2x+d$

$f(0)=0+d$

$d=2$

$f(x)=2x+2$

48. **(2)**

$$I=\int\frac{(x-x^3)^{1/3}}{x^4}dx=a\left(\frac{1}{x^2}-1\right)^b+c=\int\frac{\left(\frac{1}{x^2}-1\right)^{1/3}}{x^3}dx$$

$\frac{1}{x^2}=t,\ \frac{1}{x^3}dx=\frac{-dt}{2}$, we get

$$I=\frac{1}{2}\int t^{1/3}dt=\frac{3}{8}t^{4/3}+c=\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+c$$

$$a=\frac{-3}{8}\quad b=\frac{4}{3}\Rightarrow ab=-\frac{1}{2}\Rightarrow\frac{1}{|ab|}=2$$

49. **(a), (c)**

$f(x)=x^2+x\,g'(1)+g''(2)$ and $g(x)=x^2+xf'(2)+f''(3)$

$g'(x)=2x+g'(2),\ g''(x)=2$

$f'(x)=2x+g'(1),\ f''(x)=2$

At $x=1$ $f'(1)=2+g'(1)$ and $g'(1)=2+f'(2)$

$f'(1)=4+f'(2)$

Putting $x=2$

$f'(2) = 4 + g'(1)$

$g'(2) = 4 + f'(2)$

$g'(2) = 8 + g'(1)$

$g''(2) + f''(3) = 2 + 2 = 4$

50. **(a), (b)**

$\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$

$\sin^{-1}(1-x) = \frac{\pi}{2} - 2\theta$ $\qquad x = \sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$

$1 - x = \sin\left(\frac{\pi}{2} - 2\theta\right)$

$1 - x = \cos 2\theta$

$1 - x = 2\cos^2\theta - 1$

$1 - x = 1 - 2x^2$

$x = 0$ or $1/2$

$\alpha = 0\ \beta = \frac{1}{2}$

51. **(a), (b)**

as conjugate roots occur in pair

So $|\alpha| = |\beta|$

$|\alpha\beta| = \left|\frac{c}{a}\right|\ |\alpha\beta| > 1$

$|\alpha||\beta| > 1$

$|\alpha|^2 > 1$

$|\alpha| > 1$

52. **(a), (b)**

Vertices are equidistant from (0, 0) so origin is circumcentre (centroid)

$$\left(\frac{a(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)}{3}, \frac{a(\sin\theta_1 + \sin\theta_2 + \sin\theta_3)}{3}\right)$$

But as the centroid is the origin 0, 0

$\sum_{i=1}^{3} \cos\theta_i = 0 \qquad \sum_{i=1}^{3} \sin\theta_i = 0$

53. **(a), (b)**

Focus of the parabola $y^2 = 2px$ is $(p/2, 0)$

$\therefore$ centre of circle is $(p/2, 0)$

Radius of the circle = distance between focus and directrix

= semi latus rectum

Equation of circle

$$\left(x - \frac{p}{2}\right)^2 + (y-0)^2 = p^2 \qquad \ldots(1)$$

Solving equation (1) and $y^2 = 2px$, then

$$x^2 + \frac{p^2}{4} - px + y^2 = p^2$$

$$x^2 + \frac{p^2}{4} - px + 2px = p^2$$

$$4x^2 + 4px - 3p^2 = 0$$

$$x = p/2 \qquad x = -3p/2$$

$$y^2 = 2px = 2p\left(\frac{p}{2}\right) \quad y^2 = p^2$$

$y^2 = 2p\left(\frac{-3p}{2}\right) = -3p^2$ imaginary so not possible

$$y^2 = p^2 \quad y = \pm p$$

$\therefore$ Point of intersection are $\left(\frac{p}{2}, p\right)$ and $\left(\frac{p}{2}, -p\right)$

54. **(a), (c)**

$$g(x) = \int x^{27}\left(1 + x + x^2\right)^6 \left(6x^2 + 5x + 4\right)dx \; = \int \left(x^4 + x^5 + x^6\right)^6 \left(6x^5 + 5x^4 + 4x^3\right)dx$$

$$= x^6 + x^5 + x^4 = t$$

$$g(x) = \frac{1}{7}\left(x^4 + x^5 + x^6\right)^7 + c$$

$$g(0) = 0 \Rightarrow x = 0 \Rightarrow g(1) = \frac{3^7}{7}$$

$$g(-1) = \frac{1}{7}$$

55. **(b), (d)**

$x^a y = \lambda^a$, $(\lambda, 1)$ is a point on the given curve.

Now, differentiating equation (i) w.r.t. to *x*, we get

$$a\,x^{a-1}y + x^a\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-a\,x^{a-1}y}{x^a} = \frac{-ay}{x}$$

At $(\lambda, 1)\dfrac{dy}{dx} = \dfrac{-a}{\lambda}$

Equation of tangent at $(\lambda, 1)$

$$y - 1 = \frac{-a}{\lambda}(x - \lambda)$$

Now $x = 0$

$y = 1 + a$ $\qquad\qquad x = \dfrac{\lambda}{a} + \lambda = \dfrac{\lambda(1+a)}{a}$

$y = 0$

$$A = \frac{1}{2}(1+a)\,\frac{(1+a)\lambda}{a}$$

$$\frac{dA}{da} = \frac{1}{2}\lambda\left[\frac{a\,2(1+a) - (1+a)^2}{a^2}\right] = 0$$

$\Rightarrow\ (2a - 1 - a)(1 + a) = 0$

$(a-1)(a+1) = 0$

$a = 1 \ \ a = -1$

56. **(a), (b)**

$y\dfrac{dy}{dx} = 2y^2 \ \Rightarrow \dfrac{dy}{dx} = 2y \Rightarrow \dfrac{dy}{y} = 2dx$

$\ln y = 2x + \ln c$

$y = ce^{2x}$ Also from equation (1) $\ln y + \ln c = 2x$

$\ln y = 2x - \ln c$

57. **(b)**

$D^*(u\cdot v) = D^*[f(x)\cdot g(x)]$

$$= \lim_{h\to 0}\frac{f^2(x+h)g^2(x+h) - f^2(x)g^2(x)}{h}$$

$$= \lim_{h\to 0} f^2(x+h)\left\{\frac{g^2(x+h) - g^2(x)}{h}\right\} + g^2(x)\frac{\{f^2(x+h) - f^2(x)\}}{h}$$

$= f^2(x)D^*g(x) + g^2(x)D^*f(x)$

$= u^2\,D^*v + v^2\,D^*u$

58. **(b)**

On $f(x)=0$

$$D^*f(x)=\lim_{h\to 0}\frac{f^2(x+h)-f^2(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\lim_{h\to 0}\left(f(x+h)+f(x)\right)$$

$$=f'(x)2f(x)=0$$

59. **(b)**

If a family of n children contains exactly k boys, then by binomial distribution, its probability is

$$^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

By total probability theorem, the probability of a family of n children having exactly k boys is given by

$$\alpha p^n.{}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

Therefore the required probability is

$$=\sum_{n=k}^{\infty}\alpha p^n\,{}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}=\alpha\left(\frac{1}{2}\right)^k p^k\sum_{n=k}^{\infty}{}^nC_k\left(\frac{1}{2}\right)^{n-k}p^{n-k}$$

$$=\alpha\left(\frac{1}{2}\right)^k p^k\left[1+{}^{k+1}C_1\left(\frac{p}{2}\right)+{}^{k+2}C_2\left(\frac{p}{2}\right)^2\ldots\ldots\right]$$

$$=\alpha\left(\frac{1}{2}\right)^k p^k\left(1-\frac{p}{2}\right)^{-k+1}=2\alpha\,p^k(2-p)^{-(k+1)}$$

$$=\frac{2\alpha}{2-p}\left(\frac{p}{2-p}\right)^k\ k\geq 1$$

60. **(c)**

If A denote the event of a family including atleast one boy. Then

$$p(A)=\frac{2\alpha}{2-p}\sum_{k=1}^{\infty}\left(\frac{p}{2-p}\right)^k$$

$$=\frac{2\alpha}{2-p}\frac{\frac{p}{2-p}}{1-\left(\frac{p}{2-p}\right)}$$

$$=\frac{\alpha p}{(2-p)(1-p)}$$